Planned upgrades to existing facilities:

- adv. LIGO Plus (A+) : ↑ laser power, f-dep. squeezing (new filter cavity being installed now)
  new test masses w/ improved coating thermal noise
  - to be fully online (incl. w/ new test masses) for O5
- adv. VIRGO : phase 1 (now installing) : f-dep. squeezing ; incr. laser P to 50 W (from 20 W)
  phase 2 (for O5) : P → 500 W ; 100 kg test masses ; better coating
- KAGRA upgrades TBD

BNS Range (Mpc)

	O3 (2019-2020)	O4 (mid-2022)	O5 (2025)
			330 Mpc
LIGO	110 - 130	160 - 190	150 - 260
Virgo	50	90 - 120	130 +
KAGRA	—	25 - 130	
LIGO-India	—	—	330 Mpc

KAGRA - 3 km ; underground ; cryogenically cooled mirrors ; came online Feb. 25, 2020

recall $\mathrm{SNR} = \sqrt{4 \int_0^\infty |\tilde{h}(f)|^2 / S_n(f)\, df}$

$$= \left(\frac{1\,\mathrm{Mpc}}{D_{eff}}\right) \sqrt{4 A_{1\,\mathrm{Mpc}}^2 \int_0^\infty \frac{f^{-7/3}}{S_n(f)}\, df}$$

For optimally located (directly above / below interfer.) + oriented (face-on), $D_{eff} = D_{true}$

$$A_{1\,\mathrm{Mpc}} = -\left(\frac{5}{24\pi}\right)^{1/2} \left(\frac{G M_\odot / c^2}{1\,\mathrm{Mpc}}\right) \left(\frac{\pi G M_\odot}{c^3}\right)^{-1/6} \left(\frac{M_c}{M_\odot}\right)^{5/6} \propto M_c^{5/6}$$

Range defined as $D(\mathrm{SNR} \geq 8)$; $30 M_\odot - 30 M_\odot$ BBH , range ~ 12x BNS range

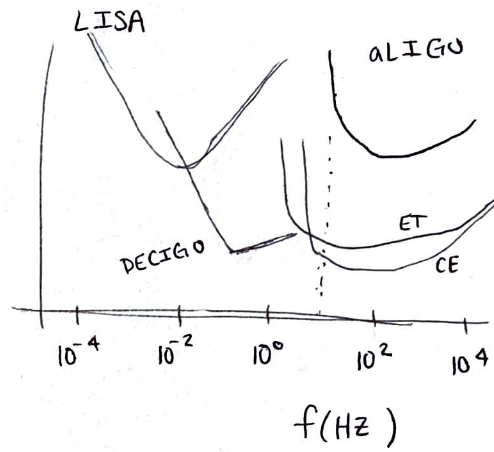
LIGO Voyager – w/in same vacuum envelope; 3x incr. to BNS range ~ ~~300~~ 1100 Mpc

- low-f cutoff → 10 Hz
- by late 2020s

current: 1064 nm

- replace glass mirrors w/ silicon optics (λ_{laser} → 1550 nm); at which Si is transparent
  ↓
  this λ requires new R&D . . .

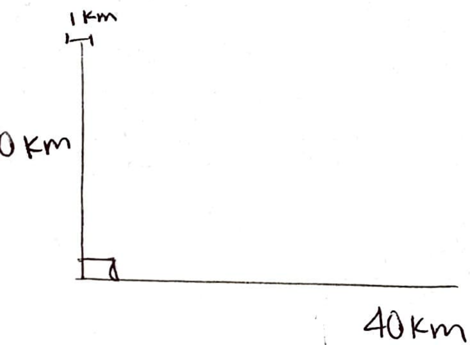
cryogenic cooling of det. at 120 K



3G Detectors

Cosmic Explorer	5 – 4000 Hz
Einstein Telescope	1 – few 1000 Hz
LISA	millihertz
DECIGO	decihertz

CE



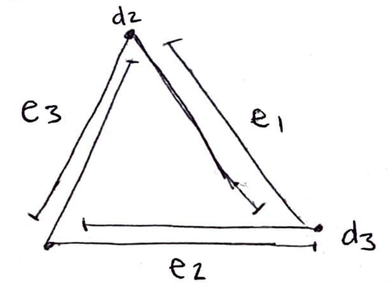
on surface; requires stable site; up to 30m Earth ust be cleared ep. on topography

two phases; main improvement comes from L arms

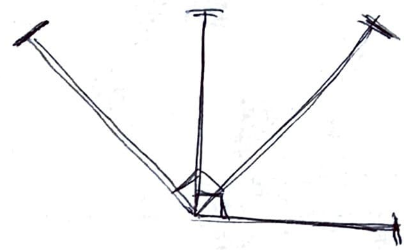
	LIGO A+	CE 1	CE 2
→ arms	4 km	40 km	40 km
→ test mass	40 kg fused silica	320 kg fused silica	320 kg silicon
suspension	0.6 m silica fibers	1.2 m silica fibers	1.2 m silicon ribbons
→ T	297 K	297 K	123 K
λ_{laser}	1 μm	1 μm	2 μm
→ P	0.8 MW	1.4 MW	2 MV
→ Squeezed light	6 dB	6 dB	10 dB
horizon redshift BNS / BBH	0.17 / 0.26	3.1 / 26	12 / 37
BNS SNR at z=0.01	150	1700	3300
BNS early warning "	10 min	40 min	90 min

② $\boxed{\text{ET}}$

3 nested Michelson interferometers



equivalent in antenna pattern & sensitivity to 2 L-shaped detectors, rot. 45° from each other w/ $3L/4$ length arms



$L = 10\,km$
$\alpha = 60°$

- triang. config: more isotropic antenna pattern; no blind spots;
  can resolve both GW pol's w/ any 2 pairs;
  fewer "end stations"

- Considering Sardinia vs. Belgium - Germany - Netherlands border

- <u>null data stream</u>:

$h^A(t) = F_+^A h_+ + F_\times^A h_\times$ be response function for each det., $A = 1, 2, 3$

antenna patterns:

$$F_+^A = d_A^{ij} e_{ij}^+ \quad , \quad F_\times^A = d_A^{ij} e_{ij}^\times$$

detector tensors defined w/ respect to unit vectors along each arm: $\vec{e}_1, \vec{e}_2, \vec{e}_3$

e.g.
$$d_1^{ij} = \tfrac{1}{2}\left(e_2^i e_3^j - e_3^i e_2^j\right)$$
$$d_2^{ij} = \tfrac{1}{2}\left(e_3^i e_1^j - e_1^i e_3^j\right)$$
$$d_3^{ij} = \tfrac{1}{2}\left(e_1^i e_2^j - e_2^i e_1^j\right)$$

$\Sigma_A h^A = 0$ for any incident rad. (any dir. or pol.)

for detector output $x^A(t) = n^A(t) + h^A(t)$,

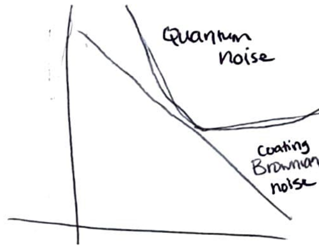
$$\sum_A x^A(t) = \Sigma_A n^A(t)$$

Sum of detector output contains only the sum of the three noise backgrounds → "null data stream"

- use to rule out spurious events, estimate noise spectral density (important for signal-dom. events), or to measure stochastic background

- unless L-shaped detectors exactly aligned by $\pi/2$, no linear comb. gives a null data ~~de~~ stream part 2

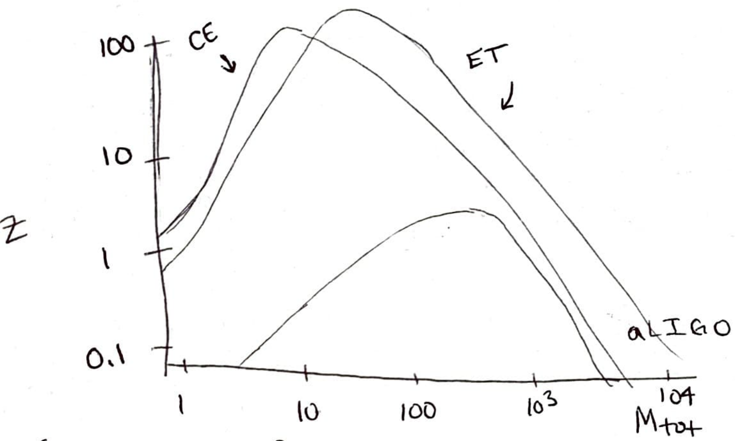
- To improve high-f sensitivity, incr. laser power (reduces shot noise);
  to reduce Brownian noise in ~~detector~~ mirror, test masses cooled to 20K
  - difficult to maintain both
  internal thermal noise
  - to remove excess heat from mirror coating, would need to increase size (thickness) of suspension fibers, which spoils performance of suspension system & ruins the low-f sensitivity



↳ "xylophone" model*: detector composed of 2 instruments in each arm

(1) LF interferometer: low power (since ~~power~~ RPN ∝ power & this dom. at low f) + cryogenically cooled mirrors → 1-250 Hz; 10K; 18 kW

(2) HF: high power + room-T mirrors → 10 Hz - 10 kHz; 3 MW



[Maggiore + 2020]

Range ~~[illegible]~~ for non-spinning, $q=1$ binaries

- BBH and BHNS w/ M_{tot} 20-100 $M_\odot$ obs. to $z \sim 20+$, probing "dark era" of universe before ~~star formation~~ first gen stars
  - BBH at such z must be primordial
- BBH ~ few $\times 10^3 M_\odot$ to $z \sim 1-5$
- BNS to $z \sim 2-3$
  (170817 was 0.01; at final design sensitivity, 2G could reach $z \sim 0.2$)

↳ $10^5 - 10^6$ BBH; 7×10^4 BNS per year for just ET acting alone

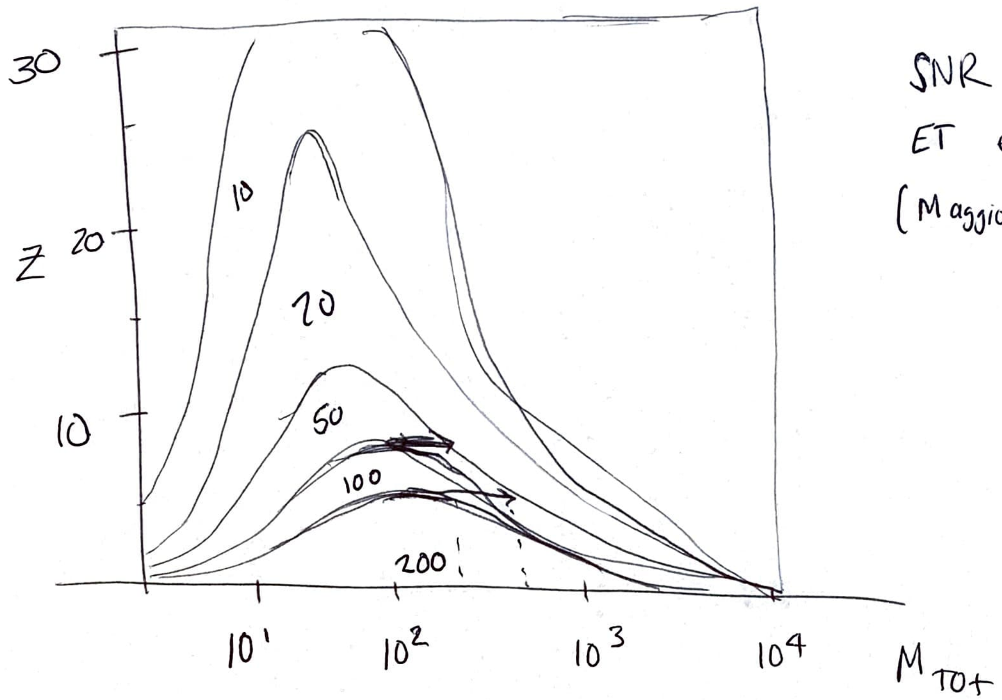
- could get $\mathcal{O}(10^2 - 10^3)$ BNS w/ EM counterparts

- BBH w/ $M_{TOT} = 50-100 M_\odot$ at $z=10$ → SNR = 50

W/ Increased Sample:

- complete pop. of BBHs → merger rate, SFR, changes in both over cosmic t, correlations w/ galaxy evolution...

- rare events: high χBBH, NS-IMBH, BBH that hasn't cleared enviro. → EM counterpar



SNR for ET events (Maggiore + 2020)

($q=1$)

Text

Concept Supported by European Commission Framework Program
Proposed to 2021 European Strategic Forum for Research Infrastructure roadmap

to begin operating: mid-2030s

[c.f. CE 1 - mid 2030s
CE 2 - 2040s]

New types of detections

[NS post-merger spectra → EOS ; peak at ~2-4 kHz]

$\boxed{\text{CWs}}$ Continuous gravitational waves (CWs)

- non-transient, nearly monochromatic ; NSs w/ "mountain" (sourced from accretion by companion), or unstable r-modes or free precession

Rigid-body theory to estimate magnitude of effect (even though NSs crusts elastic):

V = volume

$$E_{kin} = \frac{1}{2}\int \rho v^2 \, dV \quad , \qquad \vec{v} = \vec{\Omega} \times \vec{R}$$

$$v^i = \varepsilon^{ijk}\Omega_j x_k \quad ,$$

$$= \frac{1}{2}\int \rho\left[\Omega^2 r^2 - (\Omega^i x_i)^2\right] dV$$

x^i = components of position vector s.t $x_i x^i = r$

For rigid body approx, ang. velocity same everywhere:

$$= \frac{1}{2}\Omega^i \Omega^j \underbrace{\int \rho\left[r^2 \delta_{ij} - x_i x_j\right] dV}_{\equiv I_{ij}}$$

MOI tensor

$$= \frac{1}{2} I_{ij}\Omega^i \Omega^j$$

Choose axes $\vec{e}_{\hat{i}}$ that define the principal axes of the "body frame", in which $I_{\hat{i}\hat{j}}$ is <u>diagonalized</u>

then, $E_{kin} = \frac{1}{2}\left(I_{\hat{1}}\Omega_1^2 + I_{\hat{2}}\Omega_2^2 + I_{\hat{3}}\Omega_3^2\right)$

From quad. formula:

$$\frac{dE}{dt} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\right\rangle$$

Q = <u>reduced</u> quad. mom $\equiv \int \rho\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right) d^3x$ $\quad \therefore \quad Q_{ij} = -I_{ij} + \underbrace{\frac{2}{3}\delta_{ij}\int \rho r^2 d^3x}_{\text{constant}}$

$\therefore \dddot{Q}_{ij} = -\dddot{I}_{ij}$

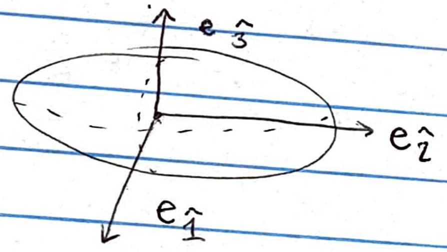
[Derivation follows from Andersson "GW Astronomy", 6.1]
book

$\boxed{\text{CWS-2}}$

So, we need I_{ij} to calculate $\ddot{I}_{ij}$, but we have $I_{\hat i \hat j}$ (i.e., in the 'body frame').

$$I_{ij} = R^{\hat j}_{j} R^{\hat k}_{k} I_{\hat j \hat k} \qquad \longleftarrow \cancel{x^i = R^i_{\hat i} x^{\hat i}} \ \text{~~and~~}$$

Consider spinning star w/ small asymm:



- Principal moments $I_{\hat 1}, I_{\hat 2}, I_{\hat 3}$ (body frame)
- rotates around $z = e_{\hat 3}$ axis
- if $I_2 \neq I_1$, not axisymmetric, will emit GWs

Then $\vec{R}$ is std rotation matrix:

$$\vec{R} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ϕ = angle btwn $\vec{e}_{\hat 1}$-axis and inertial x-axis

$\phi = \Omega t$

$$\underbrace{I_{\text{inertial}}}_{I_{ij}} = \vec{R}^T \underbrace{\vec{I}_{\text{body}}}_{I_{\hat i \hat j}} \vec{R} \qquad = \begin{pmatrix} I_{\hat 1} & & 0 \\ & I_{\hat 2} & \\ 0 & & I_{\hat 3} \end{pmatrix}$$

$$I_{xx} = \frac{1}{2} \underbrace{\left(I_{\hat 1} - I_{\hat 2}\right)}_{\equiv \Delta} \cos 2\phi \qquad \longleftarrow \qquad \frac{1}{2}\cos 2\phi = \cos^2\phi - \sin^2\phi$$

$$I_{xx} = \frac{1}{2}\Delta \cos 2\varphi = -I_{yy}$$

and $\quad I_{xy} = I_{yx} = \frac{1}{2}\sin 2\phi \cdot \Delta$

For constant rotation rate (Ω = const $\therefore \dot{\phi} = \Omega$)

$$\frac{dE}{dt} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij} \dddot{Q}^{ij} \right\rangle = \frac{G}{5c^5}\left\langle 4 \dddot{I}_{ij} \dddot{I}^{ij} \right\rangle$$

$$= \frac{32\,G}{5c^5} \Delta^2 \Omega^6$$

CWS - 3

We need to eliminate the unknown difference in MOI's, Δ.

Eqn for ellipsoid surface, assuming a uniform - ρ ellipsoid star w/ principal axes of length a_1, a_2, a_3

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} = 1$$

let $x_i = a_i \xi_i$ to tranform this to a unit sphere:

$$\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$$

For rotation about the $\hat{e}_1$- axis:

$$I_{\hat{1}} = \rho \int (x_2^2 + x_3^2)\, dx_1 dx_2 dx_3$$
$$= \rho a_1 a_2 a_3 \int (a_2^2 \xi_2^2 + a_3^2 \xi_3^2)\, d\xi_1 d\xi_2 d\xi_3$$
$$= \frac{1}{5} M (a_2^2 + a_3^2) \quad , \text{ using } V = \frac{4\pi a_1 a_2 a_3}{3}$$

Use this to re-cast Δ:

$$\Delta = I_{\hat{1}} - I_{\hat{2}} = \frac{1}{5} M \left[(a_2^2 + \cancel{a_3^2}) - (\cancel{a_3^2} + a_1^2) \right]$$
$$= \frac{1}{5} M (a_2 + a_1)(a_2 - a_1)$$

Let $\varepsilon \equiv \frac{a_2 - a_1}{(a_2 + a_1)/2}$ represent the ellipticity,

then
$$\Delta = \frac{1}{5} M \varepsilon (a_2 + a_1)^2 \cdot \left(\frac{1}{2}\right)^{-1}$$
$$= \underbrace{\frac{2}{5} M (a_2 + a_1)^2}_{} \varepsilon$$

$$\equiv I_0 = \frac{2MR^5}{5}$$; this is the MoI for a uniform density sphere w/ the same volume as our deformed star, which would in turn have radius $R^3 = a_1^2 a_2$

$\hookrightarrow \Delta = \varepsilon I_0$

Plug back into GW estimate:

$$\boxed{\frac{dE}{dt} \simeq \frac{32 G}{5 c^5} \varepsilon^2 I_0^2 \Omega^6}$$

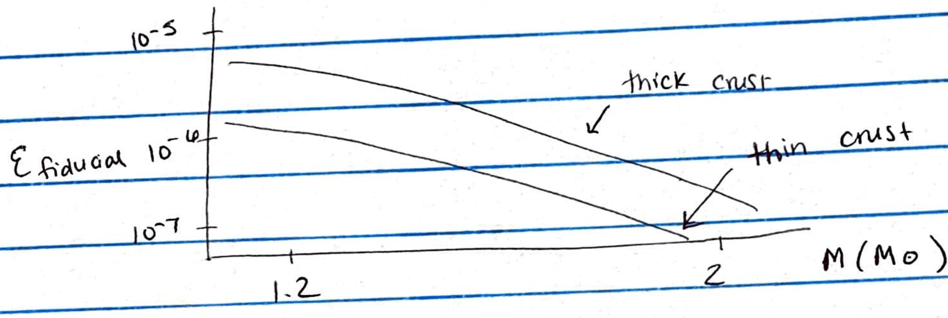
$\boxed{\text{CWs - 4}}$

Theoretical upper limit on ε:

$$\varepsilon \lesssim 2 \times 10^{-5} \left(\frac{\sigma_{break}}{0.1} \right),$$

σ_{break} = breaking strain of crust ; can be up to ~ 0.1

Maximum ε depends somewhat on mass of star, somewhat on crust models :



Physical scale of ellipticity :

If $\varepsilon \sim 10^{-6}$ on a 10 km $= 10^{6}$ cm radius star,
then typical deformation is $\sim 10^{-6} \times 10^{6}$ cm $\sim$ 1 cm

NS "mountains" typically $\lesssim$ 1 cm in height

Magnitude of strain for CW source :

$$h \approx 1.1 \times 10^{-24} \left[\frac{d}{kpc} \right]^{-1} \left[\frac{f_{GW}}{kHz} \right]^{2} \left[\frac{\varepsilon}{10^{-6}} \right] \left[\frac{I_0}{10^{45}\, g \cdot cm^2} \right]$$

- ET claims to be able to detect $\varepsilon \lesssim 10^{-9}$

CCSN

[Review: Abdikamalov + 2020 , 2010.04356]

- Shock revival :
  - ~~at~~ as hot PNS cools and contracts, it emits $\sim 10^{53}$ erg of BE as ν
    $L_\nu \sim 10^{52}$ erg/s for 10s , some of which are absorbed behind shock
  - "gain region" : ν-heating exceeds cooling → ν-driven hot bubble convection
  - in some case, standing accretion shock instability (SASI) can drive large-scale non-radial oscillations of the shock → enhances ν-heating

---

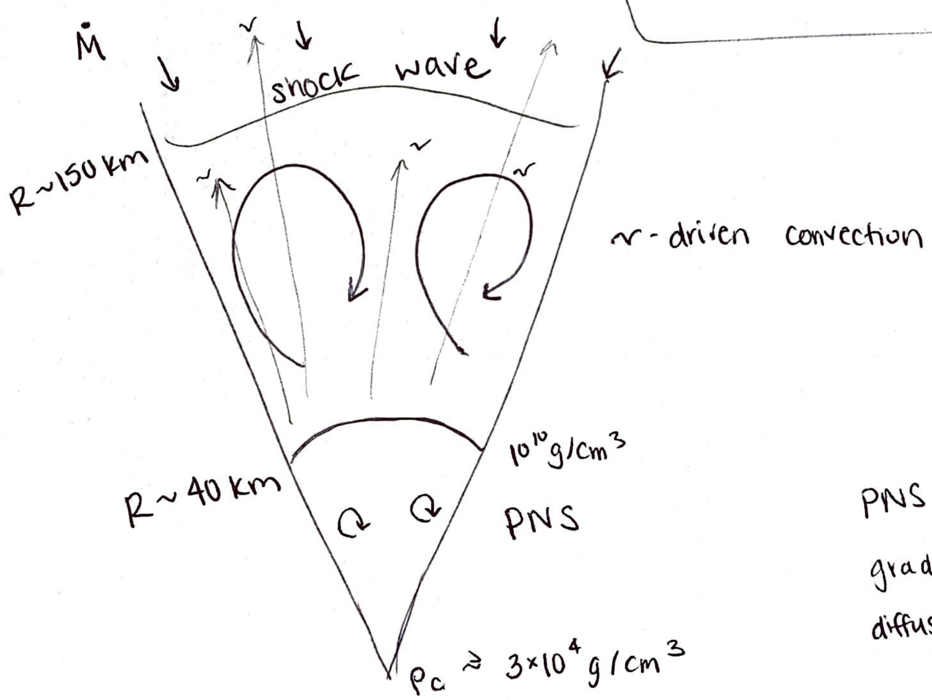
From CW derivation:
$$L_{GW} \sim \frac{G}{c^5} \varepsilon^2 I_0^2 \Omega^6$$
$$\sim \frac{G}{c^5} \varepsilon^2 (M^2 R^4)\left(\frac{v}{R}\right)^6 \qquad \& \qquad r_{sch} = \frac{GM}{c^2}$$
$$\sim \varepsilon^2 \frac{c^5}{G} \left(\frac{r_{sch}}{R}\right)^2 \left(\frac{v}{c}\right)^6$$

$R \sim$ characteristic size of source

strong GWs require : compact object, rapidly rotating, aspherical

For $\varepsilon = 0.1$, $\frac{v}{c} \sim 0.1$, $t \sim 1s$ → $E_{GW} \sim 10^{47}$ erg $\sim 10^{-7} M_\odot c^2$

→ $hD \sim 300$ cm , or $h \sim 10^{-20}$ at 10 kpc

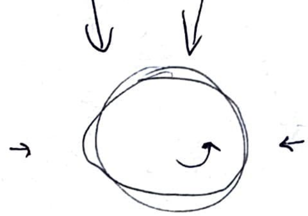


PNS convection driven by negative gradient of lepton # (from outward diffusion of νs)

- PNS convection + ν-driven conv. + SASI osc. (if appl.) ~~drive~~ excite oscillations of PNS → GWs

10a

Rotating case :



- centrifugal support → collapse is slower on equatorial plane

c.f. rot. axis

↳ PNS born w/ oblate $\ell = 2$ deformation at bounce → oscillations ~10-20ms

• degree of centrifugal deform. ~ GW strength at bounce ~ $T/|W|$

(show)

• for extreme rapid rot., centrifugal support keeps PNS from reaching very high ρ_c (i.e. not as compact), which limits GW strength

Rotating core-collapse

- Consider an oscillating, inner core, excited by Standing Accretion shock instability and convection
- rotating inner core is oblate, let

$$Q \sim M(x^2 - z^2), \quad w/ \quad z = R, \quad x = R + \delta R$$

$$\sim M\left[(R+\delta R)^2 - R^2\right]$$

Keeping terms to 1st order in δR

$$Q \sim M R \delta R$$

To estimate δR:
Impose Kepler limit on energy of rotating oblate inner core:

$$(R+\delta R)^2 \Omega^2 + \frac{GM}{R+\delta R} = \frac{GM}{R}$$

$$(R+\delta R)^3 \Omega^2 R + \cancel{RGM} = GM(\cancel{R} + \delta R)$$

For $\delta R / R \ll 1$,

$$\delta R \sim \frac{R^4 \Omega^2}{GM}$$

To estimate time derivs of Q, $\ddot{Q} = \frac{\partial^2 Q}{\partial t^2} \sim \frac{Q}{t_{dyn}^2}$

where $t_{dyn} \simeq \sqrt{\frac{R^3}{GM}}$ is usual free-fall time

$$h \sim \frac{G}{Dc^4} \ddot{Q}_{ij} \sim \frac{G}{Dc^4} M R \delta R \left(\frac{R^3}{GM}\right)^{-1}$$

$$\sim \frac{G}{Dc^4} \frac{M R^5 \Omega^2}{\cancel{GM}} \frac{\cancel{GM}}{\cancel{R^3}} \sim \frac{G}{Dc^4} G M R^2 \Omega^2$$

Plug in $T = M\Omega^2 R^2$ (kin. energy), $|W| = GM^2/R$ (grav. pot. energy)

$$h \sim \left|\frac{T}{|W|}\right| \frac{(GM)^2}{Rc^4 d} \sim 10^{-19} \left(\frac{T/|W|}{0.1}\right)\left(\frac{10 \text{ kpc}}{d}\right)$$

- linear dep. on $T/|W|$ also seen in sims

CCSN prospects:

- at 10 kpc w/ perfect knowledge of waveform, aLIGO SNR ~ 1-10 for 9-19 $M_\odot$ prog.

  ET: SNR ~ 10-100

- realistically - not guaranteed even w/ ET or CE
  - for 2G, SN range ~ our galaxy; rate for II/Ib ~ 1 per 30-100 yrs
  - ET will extend to galactic neighborhood (incl. Mag. clouds..)
    - ν-detectors (Super-K / Hyper-K / Icecube / other planned detectors) will have higher SNR than GW signal, w/ precision time res. + good sky localization
  - optimistically: ~ few / decade w/in 5 Mpc; ET horizon ~2-4 Mpc